

# **A.P. Calculus A.B.** **Summer Math Practice**

You need to work the following problems ON SEPARATE PAPER before the beginning of school. **Keep the problems in order, be neat, and show work.** DON'T WAIT until the last week of summer vacation.

You are allowed to work with your classmates and use your books, calculators, notes, and other references unless the section specifies otherwise.

Make sure that you understand this material. You have seen problems of these types in your pre-calculus class, so none of these concepts should be new.

*If there are more than forty problems in the packet that you cannot work or do not understand, you should probably get a tutor or work with a friend to insure that you are solid on these topics before school starts.*

**This packet needs to be ready for submission on the first day that we have class (the day after convocation). There WILL be some kind of grade assigned; DON'T lose it, forget to do it, etc.** In addition, at the end of the second week of school, there will be a test covering all of the material in this packet. **It is absolutely necessary that you understand these concepts before school starts.**

Good luck! See you in the fall.

I) Simplify the following fractions:

$$1) \frac{1}{x} + \frac{1}{y} =$$

$$2) \frac{1}{x} + \frac{1}{x^2} =$$

$$3) \frac{\frac{1}{x} + 1}{\frac{1}{x}} =$$

$$4) \frac{\frac{x}{x+y}}{x} =$$

$$5) \frac{3x}{3x+y} =$$

$$6) \frac{\frac{1}{x+h} - \frac{1}{x}}{h} =$$

II) Factor each expression completely:

$$7) x^2 - 16 =$$

$$8) x^2 - x - 6 =$$

$$9) 6x^2 - x - 2 =$$

$$10) 4x^3 - 19x^2 - 5x =$$

$$11) x^2 + 9 =$$

$$12) x^4 - 13x^2 - 30 =$$

$$13) x^3 + 27 =$$

$$14) x^3 - 8 =$$

$$15) (2x - 3)^3 (x + 1) + (x - 3)(2x - 3)^2 =$$

$$16) (3x - 2)^{-4} (x + 3) + (x + 3)^2 (3x - 2)^{-3} =$$

III) Solve each of the following equations for x:

$$17) x^2 + 5x - 24 = 0$$

$$18) x^2 - 9 = 5$$

$$19) x^3 - x^2 - x = 0$$

$$20) 3x^2 - 5x - 2 = 0$$

$$21) x^2 - 4x = 0$$

$$22) (x - 1)(x^2 - 11x + 30) = 0$$

$$23) \sqrt{x} + 1 = 41$$

$$24) \frac{y}{x+1} = \frac{z}{x}$$

$$25) \sqrt[3]{x+1} - 4 = -1$$

$$26) \log_5 x = 2$$

$$27) x^{-2} = \frac{1}{9}$$

$$28) \log_3 \frac{1}{81} = x$$

$$29) 2\sqrt{x} = x - 3$$

$$30) \log_8 x = \frac{-1}{3}$$

$$31) \frac{8+x}{x} - 5 = 0$$

$$32) x^{-1} = -3$$

$$33) x^{\frac{4}{3}} = 81$$

$$34) x^2 + x - 1 = 0$$

35)  $\sqrt{x} = \sqrt[3]{x}$

36)  $(x-3)^2 + 9 = 25$

37)  $3x^2 - 6x - 24 = 0$

38)  $\frac{2x-1}{(x+2)(x^2+3)} = 0$

39)  $x^3 - 2x^2 - 5x + 6 = 0$

40)  $\frac{x}{x+2} - \frac{2}{2x-1} = \frac{1}{5}$

41)  $xy - z = z + wx - 3x$

42)  $\frac{4x}{x^2+3} - 1 = 0$

43)  $(2x-1)^2(x-5)^2 + (2x-1)^3(x-5) = 0$

IV) Simplify the following as much as possible:

44)  $2 \log 2 =$

45)  $\frac{\log 8}{\log 2} =$

46)  $\log 8 - \log 2 =$

47)  $\log 4 + \log 1 =$

48)  $\log 4 \bullet \log 1 =$

49)  $(\log 2)^2$

V) Write an equation of the line based on the given information:

50) Slope of 5, passing through (3, -4)

51) Passing through (4,1) and (3,-2)

52) Through (-2,1) and parallel to  
 $4x + 2y = -1$ 53) Slope of 0, passing  
through (-5,1)54) Through the origin and perpendicular  
to  $3x + 4y = -7$ 55) Undefined slope, passing  
through (4, -5)

56) x-intercept of 5 and y-intercept of 3

57) Vertical, passing through (3,2)

58) Horizontal line passing through (1,-5)

VI) Find the intercepts:

59) Find the x and y-intercepts of the line that passes through the point (3, -5) and is perpendicular to the line  $x - 3y = -2$ 60) Find the x and y intercepts of  $\frac{x^2}{4} - \frac{y^2}{9} = 1$ .

61) Find the x and y intercepts of  $\frac{y^2}{4} + \frac{x^2}{16} = 1$

62) Find the x and y intercepts of  $x^2 + y^2 = 9$ .

VII) Sketch the following graphs (Calculator Hostile):

63)  $y = 3x + 1$

64)  $y = x^2$

65)  $y = |x|$

66)  $y = x^3$

67)  $y = -4$

68)  $x = 3$

69)  $y = \ln x$

70)  $y = \sqrt{x}$

71)  $y = \frac{1}{x}$

72)  $y = \frac{1}{x^2}$

73)  $y = |x + 1|$

74)  $y = x^2 + 2x - 3$

75)  $y = x^3 + 1$

76)  $y = (x + 1)^2$

77)  $y = 1 - x^2$

78)  $y = (x + 1)^{\frac{1}{3}}$

79)  $y = x^{\frac{2}{3}}$

80)  $y = (x + 1)^{\frac{-1}{2}}$

81)  $y = \log x$

82)  $y = e^x$

83)  $y = \frac{-1}{x}$

84)  $x^2 + y^2 = 25$

85)  $\frac{x^2}{9} + \frac{y^2}{4} = 1$

86)  $\frac{x^2}{16} - \frac{y^2}{4} = 1$

87)  $y = \begin{cases} -2 & \text{for } x < -1 \\ x & \text{for } -1 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$

88)  $y = \begin{cases} x + 1 & \text{for } x < 1 \\ x^2 & \text{for } x \geq 1 \end{cases}$

VIII) Rewrite the following absolute value functions as piece-wise functions.

$$89) y = |x|$$

$$90) y = |x - 1|$$

$$91) y = |x + 3| + 1$$

$$92) y = \frac{|x| - 2}{x - 2}$$

IX) Find the domain and range of each function:

$$93) y = x - 1$$

$$94) y = \frac{1}{x}$$

$$95) y = \frac{1}{x^2 + 1}$$

$$96) y = \frac{1}{\sqrt{x^2 + 2x + 1}}$$

$$97) y = \sqrt{x - 4}$$

$$98) y = |x - 1| + 2$$

$$99) y = \log x$$

$$100) y = 2^x$$

$$101) y = \sqrt{x^2 - 3x - 4}$$

$$102) y = \begin{cases} x + 3 & \text{for } x < 2 \\ x^2 & \text{for } x > 2 \end{cases}$$

X) Find the inverse of each function:

$$103) f(x) = x + 3$$

$$104) f(x) = \sqrt{x}$$

$$105) f(x) = \frac{x}{x + 2}$$

$$106) f(x) = 3^x$$

$$107) f(x) = \log_2(x - 3)$$

$$108) f(x) = \frac{1}{x - 3}$$

XI) Find the compositions of the function if

$$f(x) = x^3 + 1, \quad g(x) = x^2 - 2, \quad \text{and} \quad h(x) = x + 3$$

$$109) f(2) =$$

$$110) f(h(x)) =$$

$$111) f(g(2)) =$$

$$112) g(g(x)) =$$

$$113) g(f(x)) =$$

$$114) h(g(x)) =$$

$$115) h(x^2) =$$

$$116) f(x + h) =$$

$$117) h(p) =$$

$$118) \frac{g(t + h) - g(t)}{h} =$$

$$119) \frac{g(x + h) - g(x)}{h} =$$

XII) Solve the system of equations:

$$120) \begin{cases} 2x + 3y = 8 \\ x + 2y = 5 \end{cases}$$

$$121) \begin{cases} y = x^2 + 2x + 9 \\ 7x + y = 19 \end{cases}$$

XIII) Intersection of Curves:

122) Find the intersection of the lines  $y = x + 1$  and  $3y - x = 5$ .

123) Find the points of intersection between  $y = x + 7$  and  $y = x^2 + 2x + 5$ . Also sketch the area between the graphs.

XIV) Write the following formulae:

124) Quadratic formula

125) Pythagorean theorem

126) The hypotenuse of a 45-45-90 isosceles right triangle with a leg of length  $x$ .

127) The hypotenuse of a 30-60-90 right triangle with the shorter leg having a length of  $x$ .

128) The hypotenuse of a 30-60-90 right triangle with the longer leg having a length of  $x$ .

129) The volume of a cylinder

130) The volume of a cone

131) The volume of a sphere

132) The volume of a box with a square base.

133) The surface area of a sphere

134) The lateral area of a cylinder

135) The surface area of a cylinder

136) The area of a triangle

137) The area of a trapezoid

138) The cross section through the center of a sphere

139) The volume of a triangular prism with a height of  $y$  that has an equilateral triangle with sides of length  $x$  as the base.

XV) Solve by using similar triangles:

140) A six-foot tall man is standing 10 feet away from a 20-foot light pole. What is the length of the man's shadow?

141) Water is dripping out of the bottom of a cone that has a diameter of 8 inches and a height of 12 inches. When the depth of the water in the cone is 8 inches, what is the radius of the water?

XVI) Describe the symmetry of the following functions (Even, Odd, or Neither):

142)  $y = x^2 + 1$

143)  $y = x^3 + 1$

144)  $y = |x|$

145)  $y = x^3 + x$

146)  $y = \sin x$

147)  $y = \cos x$

148)  $y = x^2 \sin x$

149)  $y = x^3 \csc x$

150)  $y = \tan x$

XVII) Find the equations for the horizontal and vertical asymptotes of each function.

151)  $y = \frac{1}{x-1}$

152)  $y = \frac{x}{\sqrt{x^2-9}}$

153)  $y = \frac{x^3}{x^3-1}$

154)  $y = \frac{x}{\sqrt{x^2+1}}$

155)  $y = \frac{x^2-4}{x+1}$

156)  $y = \frac{x^2-x-6}{2x^2+6}$

XVIII) Evaluate the following:

157)  $\sum_{k=0}^{\infty} (12) \left(\frac{2}{3}\right)^k =$

158)  $\sum_{k=12}^{254} 2k + 4 =$

XIX) Exponent Rules: Which of the following are true?

159)  $x^0 = 1$

160)  $x^{-2} = \frac{1}{x^2}$

161)  $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

162)  $x^5 \cdot x^3 = x^{15}$

163)  $x^5 \cdot y^5 = (xy)^5$

164)  $(x^3)^5 = x^8$

165)  $x^{5-w} = \frac{x^5}{x^w}$

166)  $x^{t+s} = (x^t)^s$

167)  $\sqrt{\frac{9}{4}} = \sqrt{\frac{3}{2}}$

168)  $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$

169)  $\sqrt{x^2} = |x|$

170)  $\sqrt{x^2 - 1} = x - 1$

171)  $x^{\frac{4}{3}} = \sqrt[4]{x^3}$

172)  $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$

173)  $x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}}$

174)  $e^{\ln x^2} = x^2$

175)  $\ln e^3 = 3$

176)  $e^{2\ln 2 - \ln 5} = \frac{4}{5}$

177)  $\ln x^2 = (\ln x)^2$

XX) What are the following trigonometric identities:

178)  $\sec x =$

179)  $\csc x =$

180)  $\tan x =$

181)  $\cot x =$

182)  $\cos^2 x - 1 =$

183)  $\sec^2 x - 1 =$

184)  $\cot^2 x + 1 =$

185)  $\cos^2 x$  (half-angle) =

186)  $\sin(a + b) =$

187)  $\sin^2 x$  (half-angle) =

188)  $\sin 2x =$

189)  $\cos(a + b)$

190)  $\cos 2x$  (list all three equations)=

XXI) Find the period of the following functions (Calculator Hostile):

$$191) y = 4 \cos\left(\frac{2\pi}{365}(x - 80)\right) + 15$$

$$192) y = \frac{2}{3} \cos(2x) - 12$$

$$193) y = 12 \sin\left(\frac{2\pi}{12.4}(x - 3)\right) + 15$$

$$194) y = 44 \tan\left(\frac{x}{3}\right)$$

XXII) Evaluate the following expressions (Calculator Hostile): Dig out your Unit Circle.

$$195) \sin\left(\frac{\pi}{6}\right) =$$

$$196) \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$$

$$197) \tan(210) =$$

$$198) \cos(0) =$$

$$199) \cos(45) =$$

$$200) \csc\left(\frac{-5\pi}{6}\right) =$$

$$201) \sec(180) =$$

$$202) \cot\left(\frac{-\pi}{2}\right) =$$

$$203) \sin^{-1}\left(\frac{-1}{2}\right) =$$

$$204) \tan\left(\frac{\pi}{4}\right) =$$

$$205) \sin^2\left(\frac{5\pi}{6}\right) =$$

$$206) \cot\left(\frac{2\pi}{3}\right) =$$

$$207) \sin\left(\frac{\pi}{2}\right) =$$

$$208) \cos^{-1}(-1) =$$

$$209) \sec\left(\frac{3\pi}{4}\right) =$$

$$210) \tan^{-1}(-1) =$$

$$211) \csc(180) =$$

$$212) \sec^2\left(\frac{11\pi}{6}\right) =$$

$$213) \cos\left(\tan^{-1}\left(\frac{-12}{5}\right)\right) =$$

$$214) \csc\left(\cos^{-1}\left(\frac{-2}{3}\right)\right) =$$

$$215) \sin(\tan^{-1}(\cos \pi)) =$$

XXIII) Sketch one or two periods of the following trigonometric graphs (Calculator Hostile):

$$216) y = \sin x$$

$$217) y = \cos x$$

$$218) y = \tan x$$

$$219) y = \cot x$$

$$220) y = \sec x$$

$$221) y = \csc x$$

$$222) y = \sin(2x)$$

$$223) y = 3\cos(3x) - 2$$

$$224) y = |\sin x|$$

$$225) y = 4\cos\left(\frac{\pi}{6}(x-1)\right) + 4$$

$$226) y = \cos\left(\frac{\pi}{4}x\right) + 3$$

$$227) y = 2\sec\left(\frac{1}{2}x\right)$$

XXIV) Solve the following trigonometric equations for the given domain (Calculator Hostile):

$$228) \sin x = \cos x \quad ; \quad [0, 2\pi] \quad 229) \sin 2x = \frac{1}{2} \quad ; \quad [0, 2\pi]$$

$$230) \sin 3x = \frac{\sqrt{3}}{2} \quad ; \quad [0, 2\pi] \quad 231) \cos 4x = \frac{-1}{2} \quad ; \quad [0, 2\pi]$$

$$232) \tan 2x = 0 \quad ; \quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \quad 233) \sin^2 x - \sin x = 0 \quad ; \quad \text{for all } x$$

$$234) \cos 2x = \cos x + 2; \quad [0, 2\pi] \quad 235) \cos\left(x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2} \quad ; \quad [0, 2\pi]$$

$$236) \tan^2 x = 3 \quad ; \quad [0, 4\pi] \quad 237) \sec^2 x - 1 = \tan x \quad ; \quad [0, 4\pi]$$

XXV) Use the graphing calculator to answer a few random questions:

238) Sketch  $y = 0.1x^3 + 2x^2 - x - 3$  in a reasonable viewing window.

239) Find the roots (x-int) of the graph in prob #238.

240) Find the point(s) of intersection for  $y = x^3 + x - 3$  and  $y = 2x + 4$ .

241) Given  $f(x) = -x^4 + x - 4$ , find the maximum value of the function, and find the interval(s) on which  $f(x)$  is increasing.