In the second unit on waves and optics we finally turn our full attention to light. In this particular unit we will tend to avoid the wave nature of light while emphasizing the ability of light to travel in a straight line except at certain boundaries. At these boundaries light may reflect or refract or both. Before taking the test on unit 305 be sure that you are fully aware of each of the topics listed in the following outline.

I. Reflection and Transmission at a Flat Boundary

A. Reflection

B. Refraction

1. **Index of Refraction**
2. **Snell's Law**
3. **Total Internal Reflection**
4. **Dispersion**

II. Image Formation with Spherical Mirrors

A. Ray Tracing

B. Algebraic Approach

C. Special Cases

III. Image Formation with Thin Lenses

A. Ray Tracing

B. Algebraic Approach

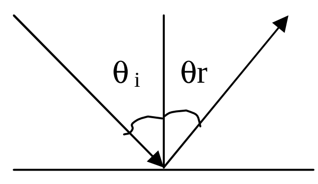
Lesson 3-28 **Electromagnetic Spectrum, Reflection**
Read Sections 13:1 – 2 and 14:1 **and Index of Refraction**

Often light travels through a vacuum. All light waves travel at the same speed in empty space. We designate the speed of light in a vacuum with letter “c”. Let c = 3.0E+8 m/s and store this value in your calculator in the alpha symbol. Most physicists and astronomers think of light as more than what you can see with your eyes. Generally they refer to the electromagnetic spectrum discussed on pages 737-738. To them, radio waves to visible light to x-rays are all pretty much variations on the same theme, oscillating electromagnetic fields or light. Thank you Mr. Maxwell.

$$c = f \lambda$$

All parts of the electromagnetic spectrum including gamma rays will behave according to the equation in the box to the right while traveling through empty space. Do not be tricked into picking one form over another when asked which moves at the fastest speed in a vacuum? Some waves do vibrate more slowly as compared to others. In Table 1 on page 447 the slower vibrations are at the top of the chart while the faster vibrations are at the bottom. As you move down the chart in increasing frequency you will see the wavelength getting shorter. This is because the product of f and λ must always result in the same value of “c”, the speed of light in a vacuum. In order for that to happen as one value gets larger the other value must shrink. Be sure that you **know the entire order of the spectrum in Table 1** according to longest to shortest wavelength (λ), lowest to highest frequency(f), fastest to slowest speed (trick question) and from lowest energy to highest energy (same as frequency). The only numbers that you should be familiar with are the wavelengths of visible light. For ROY G BV the wavelength at the red end of the visible spectrum is around 700 nanometers. At the violet end of the spectrum the wavelengths are around 400 nanometers. For the rest of this course light is a synonym for the entire electromagnetic spectrum unless otherwise stated.

Homework Problems **Practice A page 449.**



Law of Reflection

In this course we consider only very smooth surfaces. The law of reflections as shown in figure 13.7 is simply stated, “Angle in equals angle out”. But you must make note of a peculiarity. Angles in optics are measured from the normal to the surface instead of from the surface. In general, an “i” subscript is notation for the *incident* ray. An “r” subscript is for the *reflected* ray.

$$\theta i = \theta r$$

Homework Problem **none**

Index of Refraction

We have just mentioned that the speed of light in a vacuum is the same for all wavelengths. But what happens when light passes through a transparent substance? Since there are now atoms in the way the light must change speed. What will this do to frequency and wavelength? How do you compare the speed of light through one medium as compared to another? Consider light moving through water and glass for example. The central key to all of these concerns is the index of refraction.

$$n \equiv c/v$$

The index of refraction, "n", is defined as the ratio of speed of light in a vacuum to speed in a medium. Because of the way it is defined the value of the index for any material will be $1 < n < \infty$. If n = 2 then the speed of light is half the normal speed. Values of n are listed in Table 14.1 on page 490. Of course the authors will mention names of materials and expect you to use the table to get the corresponding value for the index of refraction. Unless otherwise stipulated you can assume that air is approximately a vacuum. Compare the value of n for air to that of a vacuum. The speed of light in water can be found using the value of n=1.333 for water. v = c/n = 3E8/1.333 = 2.25E+8m/s

So we recognize that when light goes from a vacuum to water it has a change in speed. What about the frequency? This is a very critical question. The answer is that frequency is constant.

When going from one medium to another light crosses a boundary. We call this *transmission*. Recall the prefix "trans" means to cross. When crossing a boundary the speed of light changes but frequency does not. This has the affect on wavelength that if speed decreases then so does wavelength and if speed increases so does wavelength. See figures 14.3 on page 489.

Example Problem

Assume that light in medium 1 is moving through air with a wavelength of 500 nm. Suppose that medium 2 is a piece of glass with n=1.50. Find the speed, frequency and wavelength on both sides of the boundary.

In Air

$v \cong c$ = 3E8 m/s

λ = 500 nm as given

$f = c/\lambda$ = 6 E+14 hz

In Glass

v = c/n = 3E8m/s/1.5 = 2.0E8m/s

f = 6E+14Hz, same as before

λ= v/f = 2E8m/s / 6E14hz = 333 nm

Homework Problems **Practice A page 493**

Compare and Contrasting Waves

In this lesson we started by comparing two different waves in the same medium, a vacuum. When you compare two waves in a vacuum the speeds are the same and the frequency and wavelength will vary inversely.

We ended this lesson by comparing one wave in two different mediums. In this case the frequency remains the same for the single wave across the boundary but the speed and wavelengths will change directly. Some students tend to confuse the two comparisons in an attempt to blend them into a single thought. Do not attempt this. These are two different ideas.

In today's lesson we considered what happens when a single wave strikes a boundary at an angle of 0° incidence. We did this in order to focus on the values of speed, frequency and wavelength as the wave crosses the boundary. What happens if the wave is not normal to the surface? What if we have a problem as shown in figure 14.3? How does the angle change across a boundary? That is tomorrow's lesson.

Lesson 3-29 **Snell's Law, Dispersion & Internal Reflection**
Read Sections 14:1-2

In figure 14.2a, a beam of light is shown striking the boundary between air and glass. Part of the light reflects and part of the light transmits. The amount of light that reflects and transmits can be calculated but is not included in this course. The direction of the reflected and transmitted beam is of concern. They are shown in figure 14.2a. The text labels the incident angle as θ_1 but we will designate the incident angle as θ_i. The book calls the reflected angle θ_1 but we will use θ_r. The book calls the transmission angle θ_2 but we choose θ_t. How are the incident and transmission angles related? This is described by Snell's Law and is shown in the box to the right.

$$n_i \sin \theta_i = n_t \sin \theta_t$$

The angles are related to speed. The critical idea is to be able to explain why the beam either bends towards the normal or away from the normal. Both are shown in figures 14.2 a & b. Notice in figure 14.3a the beam slows down going from air to water and bends towards the normal. Christian Huygens explained this in the following manner. The bottom part of the wave hits the water first and slows while the upper half of the wave is still moving at a faster speed in air. The beam bends to the side that drags first as shown in figure 14.3. In figure 14.2 b the light is increasing speed going from glass to air. When light increases speed across the boundary it will bend away from the normal because the top half of the wave will increase speed before the bottom half. Be sure that you fully understand the bending in and out from a physical point of view and can explain it in your own words to others. See example problem 14.A for application of Snell's Law.

Homework Problems **Practice A page 493**.

Dispersion (Note the root word here is disperse.)
Up to this point we have considered only monochromatic (one color) sources of light like a laser. It so happens that the index of refraction is not the same exact value for all parts of ROYGBV. This is why Table 14.1 has specifically stated "measured at 589nm". If you look at table 26.2 you will notice that the indices of refraction have a slight variation in value across the visible spectrum. Crown glass for example goes from a value of n=1.520 at the red end of the spectrum up to a value of 1.538 at the violet end of the spectrum. As a result of variation on indexes of refraction when white light strikes a transparent material at an angle the different colors bend to different angles. The spreading of white light into its colors is known as *dispersion*.
Homework Problems **Read section 14:3**

Total Internal Reflection
When light passes across a boundary where it can increase speed **($n_t < n_i$)** total internal reflection is possible. As you can see in figure 26.10 a, the light is bending away from the normal. This means that the transmission angle will reach 90° before the incident angle (14.10b). When the transmission angle is at exactly 90° the incident angle is said to be at the "critical angle". Any light approaching the boundary at the critical angle or at an angle higher than the critical angle cannot escape the incident medium. All rays at the

critical angle or greater experience total internal reflection. The critical angle is found by taking Snell's Law, allowing $\theta_t = 90°$ and solving for the incident angle. The result is highlighted in equation [26.4] on page 790. As a result of this effect a perfectly transparent material can be made into a mirror as shown in figure 14.12.

Homework Problems **Practice 14 C page 508**

Lesson 3-30 (2 days) **Image Formation with Spherical Mirrors**
Read Sections 13:2 – 4

When an object is placed in front of a mirror many rays of light will be emitted from the surface of the object. A curved reflecting surface can be used to collect many of these rays and form an image of the original object. Figure on p. 461 shows three of the many rays coming from a point at O and meeting again at point I. The purpose of this lesson is to explore how a curved mirror forms an image using several methods.

Spherical Mirrors and Focal Length

Coating the outside of a perfect sphere with a reflective material makes a spherical, convex mirror. If the sphere is hollow the inside of the sphere can be coated to form a concave mirror. The entire sphere does not have to be used; a mere slice will suffice to form an image (Tables 13:4&5). Your text shows two-dimensional drawings. In either the convex or the concave case the curve is based on a radius from the center of the original sphere to the surface. The letter C in all diagrams represents the center of the original sphere. The distance from C to the mirror is the radius. If parallel rays of light shine upon a spherical surface they will tend to merge at a single point called the focal point, "*f*". The focal point is always half the radius of the sphere. This is shown directly in figure 13.12 and drawn schematically in figure 13.12. Note that for convex mirrors all of the rays will diverge (spread out) upon reflection at the surface but they will tend to spread from a common focal point behind the mirror.

f = radius/2

This brings us to our first note of sign convention. **If the focal point is on the reflecting side of a mirror it has a positive length. If the focal length is on the non-reflecting side of a mirror it has a negative focal length.** This is because mirrors should reflect, not transmit. Convex mirrors have $-f$ (see Table 13-5) while concave mirrors have $+f$ (Table 13-4).

Ray Tracing

Any object can be placed in front of a mirror and used for image formation. On page 455 the object is a toy. Both toy and image are shown in the picture. In ray diagrams an upright arrow represents the object, no matter what. Since the base of the object is on the principle axis of the mirror then the base of the image will also be on the principle axis of the mirror. **Using rays from the highest point of the object (arrow tip) to locate the highest point of the image helps us fill in the rest of the image between tip and principle axis.** There are an infinite number of rays leaving the tip of the object. It would take a long time to draw all of them. Fortunately for us we only need to know how to draw two to four rays in order to establish where the image is formed. The method of using these rays in order to establish the location of the image is known as *ray tracing*. Examples of ray tracing are shown in Tables 13:4 and 13:5

The Four Rays (sound like a bad song group from the 60's?)

The book lists three rays that are easily drawn. I will add a fourth to their list on page 459 and also expand their description. WARNING! Do not attempt to learn all four rays at once. Read each description below, then trace that ray through figures in Table 4 & 5.

- Ray 1 ([1]) leaves the tip of the object going parallel to the principle axis towards the mirror. Upon reflection, the ray travels along a line referenced through the focus. Notice for fig13.13 the ray does not pass through the focal point because f is behind the mirror. The reflected path, however, is aligned with the focal point.
- Ray 2 ([2]) leaves the tip of the object aligned with the focal point but headed towards the mirror. Upon reflection this ray will be parallel to the principle axis. On page 461 this ray passes through f. In Table 4-6, ray 2 never crosses f because the object is closer to the mirror than the focal point. In fig13.13 the ray encounters the mirror before reaching f. In two of their three examples the ray does not go through the focal point as their description implies. This is why "aligned with" is a better way to think of ray 2.
- Ray 3 ([3]) leaves the tip of the object and approaches the mirror aligned with the center, C, of the curvature of the mirror. In some cases the ray passes through C before getting to the mirror, in other cases the ray only encounters C after reflection. In fig13.13 the ray never gets to C because that point is located behind the mirror. Again the phrase "through the center" is a poor choice of words.
- Ray 4 (invisible?) leaves the tip of the object and goes directly to the intersection of the principle axis and the mirror. Upon reflection the ray forms the same angle below the principle axis going outward that it had formed between the principle axis and inward ray. This is my favorite ray because it does not matter if the mirror is concave or convex it is always drawn the same way. If the tip of the object is 3 cm above the principle axis then the reflected part of ray 4 will pass through a point 3cm below the axis at the object. It is difficult to introduce error in this ray as well.

If the rays, after reflection, converge to a point on the reflected side of the mirror you have a ***real image***. The image will also be inverted. If the rays seem to diverge after reflection then you have a ***virtual image*** that will be upright. In order to locate the place to draw the virtual image you must trace the reflected rays back to their common point of reflection that is behind the mirror. Notice the dark, dashed lines in fig 13.13 and Table 4-6. All images can be classified as either real or virtual. They can also be classified as upright or inverted. Finally they can be classified as enlarged or reduced. The image in Table 4-4 is real, inverted and enlarged. The image in Table 4-6 is virtual, upright and enlarged. Make-up mirrors work this way. The image in 13.13 is virtual, upright and reduced. This will be true of all convex mirrors including side view mirrors. Flat mirrors produce upright, virtual images that are the same size. You can see both real and virtual images. Only real images can be projected onto a screen or other surface.

Algebraic Approach

Let the distance that an object is from the mirror be the object distance, d_o. Let the distance the image is from the mirror be d_i. The focal length is f. All of these values are positive if they are measured on the reflecting side of the mirror and negative if on the transmitting side of the mirror.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The three values are related as expressed in the above text box. This is a great place for using the [x^{-1}] button on your calculator.

✓ **If image distance is positive the image is real; negative d_i implies a virtual image.**

There is another equation also. The magnification of the image can also be determined algebraically. The true definition of magnification is the ratio of image height to object height. We can also use the object and image distances to find magnification.

$$M = \frac{h_i}{h_o} \text{ or } M = \frac{-d_i}{d_o}$$

✓ **A positive M means that the image is upright.**

✓ **If absolute value of |M| is > 1 object is enlarged; if |M| < 1 image is reduced.**

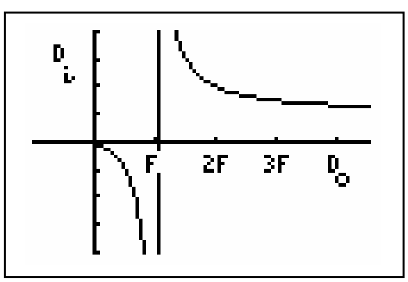
Homework Problems **Practice B page 462 and Practice C page 466 Use Algebraic approach for all problems**

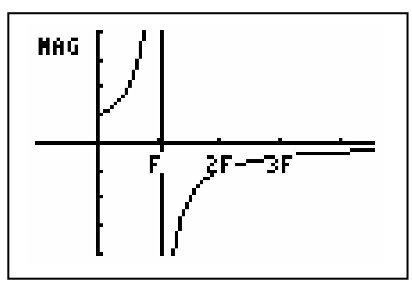
Graphical Analysis and the TI 83+

Imagine that you stand x meters in front of a mirror with a focal length of f = +1 unit. Your image is formed a distance of Y_1 from the mirror. Make a graph of your image distance as a function of your actual distance from the mirror. What happens to your image as you walk towards the mirror from very far away? Let your Magnification be Y_2. What does your magnification look like as a function of distance from the mirror? Use the upper boxed equation on this page and let d_o=x, d_i = $Y_1$and f=1. Solve for Y_1 = x/(x-1)/(x≥0). If you graph this with a window that is –1<x<5 and –5<Y_1<5 you will see a graph that is negative inside of the focal length and positive outside the focal length. Standing in front of a concave mirror can produce real and virtual images depending upon where the object is located. If you repeat the same exercise for a convex mirror with f = –1 you will find that no matter where you stand the image distance is negative so that the image is always virtual.

This graphing exercise can be extended to graph the magnification as a function of distance from the mirror. Do not erase Y_1 but do turn it off by un-highlighting the equal sign. Go down to Y_2 and have the calculator graph Y_2 = -Y_1/x. This will give you the magnification as a function of distance in order that you can tell where you get enlarged or reduced images as well as inverted or upright. Graphs are shown below for the concave mirror case.

Image distance= f(object distance) Magnification = f(object distance)





You can see that the magnification is positive for values of x less than the focal length. So the image is upright if you are inside of the focal length and inverted outside of focal length. While standing outside of focal length your magnification is enlarged between f and C. Your image is reduced for any distance greater than C from the mirror.

Lesson 3-31 (2 days) **Image Formation with Thin Lens**
Read Sections 14:2-3

The ideas of today's lesson are very similar to the previous lesson with a few exceptions. You can form images that are real or virtual, inverted or upright and so forth with either spherical mirrors or with thin lenses. There are very few contrasts. Lenses form images by transmission and refraction while mirrors use reflections. Mirrors will focus all wavelengths to the same point while lenses will bend different wavelengths to different angles (dispersion troubles). The focal length of a mirror is simply half of the radius. The focal length of a lens is determined by the curvature of both front and back faces of the lens as well as the indices of refraction of the lens and its surrounding medium. A mirror has one focal point while a lens will have two focal points. The positive for a mirror is the reflecting side because mirrors reflect. The positive side for a lens is the transmission side because lenses transmit light. There are three rays for lenses while there are 4 easy rays for mirrors.

The lessons have the following in common. You can analyze image formation using ray tracing techniques or by an algebraic method. The same equations from yesterday's lesson work today as well but beware that positive and negative signs have been reversed for d_i. A positive image distance is transmitted.

Focal length of a lens

Figure 14.5a shows how rays for a convex lens will focus parallel rays to a point on the transmission side of the lens. Another name for a double convex lens (fig 14.5a) is a convergent lens since it will make the rays converge. The bigger the difference between the index of refraction of the lens and the surrounding medium the quicker the rays will converge and the smaller the value of f. If the lens in the photograph were placed in water the yellow rays would be seen to converge farther from the lens and f would be larger in value but still positive. If the same lens were placed in a solution that had an index of refraction greater than the plastic lens then the lens would suddenly bend the rays out instead of in.

The lens in figure 14.5b is thick near the edges but very thin in the center of the lens. This type of lens is known as a concave lens. Notice how it makes the red rays diverge? Thus the other name for this type of lens is a divergent lens. Values of focal lengths for this type of lens will be negative.

Ray Tracing

Before we describe the ray tracing be sure that you have reviewed Huygens's explanation of refraction at a boundary, fig 14.3. Consider rays in figures 14.5a&b for contrast. In both instances the rays leave the object traveling parallel to principle axis. In former case the ray bends inward towards the forward focal point. In the latter case the ray bends outward? Why the difference? If you cannot explain physically why the rays bend the way they do stop and go find out before reading any more. In the first case the part of the ray closer to the center of lens gets slowed down first causing the ray to bend inward; in the latter case the outer part of the wave catches the glass first making the ray bend outward. We now describe the rays similar to page 497.

- Ray 1 ([1]) leaves the tip of the object moving parallel to the principle axis until reaching the lens. It then transmits aligned with one of the focal points. Use your comprehension of refraction to choose the correct focal point (forward or back).
- Ray 3 ([3]) leaves the top of the object and travels straight through the center of the lens without being deflected, assuming a very thin lens.
- Ray 2 ([2]) leaves the tip of the object aligned with the "other focal point" until it reaches the lens. From there is transmits parallel to the principle axis.

The cruelest part of ray tracing is that if you get ray 1 wrong you will also get ray 2 wrong and there is no way to check unless you rely upon your knowledge of refraction according to Huygens. Again note that if the rays converge on the transmission side you have a real image. If the rays tend to diverge from some point in front of the lens you will have to trace the transmitted rays backwards to locate the image as is demonstrated in Table 14:3 #6. Trace back the **second half** of the rays please. (Most common mistake made by first year students is tracing back the wrong part of the diverging rays!)

Algebraic Approach

Again the same equations are used as before just be careful with the sign conventions. Watch the vocabulary for convergent or convex lenses having a positive focal length. All divergent lenses will always form an upright, reduced virtual image. A convex lens will form either an inverted, real, reduced image or an upright, virtual image.

Homework Problems **Practice B page 501 Algebraic Method only**

This concludes our second unit on waves and optics except for a few special objects. Be totally familiar with a flat mirror where radius = ∞, $f=\infty$ and $d_i = -d_o$ with M = +1. Be totally aware that a magnifying glass works with the object (small print) inside of the focal length so that the image is virtual, upright and enlarged. A make-up mirror works with the same ideas in mind. Finally, note that there is a third way to create image formation with use of a Fresnel Lens but we have to leave that to our next unit which is Physical Optics. (Nice segue eh?)